440/640 Test 1 — Outline

Formulation of LPs
    decision variables, objective function, constraints
    standard *equality* form (min $c^T x$, $Ax = b$, $x \geq 0$)
    feasible solutions, optimal solutions
    infeasible problems, unbounded problems
    transformations to other forms: e.g., standard *inequality* forms

Algebraic Representation
    standard equality form (min $c^T x$, $Ax = b$, $x \geq 0$)
    basic variables, nonbasic variables
    matrix formulation $A = [B\ N]$, $x = [x_B\ x_N]^T$
    basic feasible solution (BFS), basic infeasible solution
$$x_B = B^{-1}b - B^{-1}Nx_N$$
    degeneracy of BFS
    optimality of BFS, reduced costs
$$z = c_B^T B^{-1}b + (c_N^T - c_B^T B^{-1}N)x_N$$

Geometric Representation
    hyperplanes, half spaces, normal vectors
    feasible region (FR) as intersection of half spaces
    corner point of FR $\rightarrow$ BFS
    unbounded objective function
    alternative optimal solutions
    degeneracy
    solution by geometric method

Simplex Method
    move from BFS x to adjacent BFS x'
    find improving direction d and step size $\alpha \geq 0$
        update $x' = x + \alpha d$
    detect unbounded problems
    tie-breaking rules
    degeneracy and cycling
        Bland's rule
    initial basic feasible solution
        Phase I LP: can detect infeasibility
    uniqueness vs. alternative optimal solutions

Duality
- defined for standard equality form
- general construction of dual LP
- properties of primal-dual pair
  - weak duality
  - strong duality
  - table of possible primal-dual outcomes
- dual variables and reduced costs
  - $u^T = c_B^T B^{-1}$
  - reduced costs ≥ 0 (primal) $\Leftrightarrow$ feasibility (dual)